THE RESEARCH DIALOGUE
An Online Quarterly Multi-Disciplinary
Peer-Reviewed / Refereed Research Journal
ISSN: 2583-438X
© The Research Dialogue | Volume-05 | Issue-02 | July-2026
Available online at: https://theresearchdialogue.com/
DOI : https://doi.org/10.64880/theresearchdialogue.v5i2.47

A Convergence-Controlled Homotopy Analysis Framework for Nonlinear Fractional Differential Equations

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Abstract

Fractional differential equations (FDEs) have gained considerable prominence in modeling scientific and engineering problems involving memory and hereditary effects. In such models, Caputo fractional derivatives of order between 0 and 1 are commonly employed to represent memory-dependent processes while preserving the applicability of classical initial conditions. The resulting dynamic models are generally nonlinear and, in most practical situations, do not admit exact analytical solutions. Because fractional derivatives provide a more realistic description of the internal dynamics of materials and complex systems, FDEs have become important for understanding biological population models, diffusion phenomena, chemical reactions, conductive heat transfer, engineering applications, and other complex processes. Consequently, the development of effective techniques for obtaining approximate solutions has become an important area of research. Fractionalization of mathematical models can be achieved through several approaches, among which the Caputo-type fractionalization has proved to be particularly effective for constructing mathematically consistent models capable of describing temporal dynamical behavior. The present work focuses on nonlinear Caputo-type FDEs and employs the Homotopy Analysis Method (HAM), a powerful analytical technique that has attracted significant attention due to its flexibility and effectiveness in handling nonlinear problems. The convergence of the fractional HAM is investigated for nonlinear Caputo fractional-order differential equations, and the existence and convergence of the obtained solutions are established within the framework of the convergence-controlled homotopy analysis method. The associated deformation equations are derived, and sufficient conditions for controlling the truncation error are obtained through the auxiliary operator, thereby improving the accuracy and reliability of the proposed approach.

Keywords: Fractional differential equations; Caputo fractional derivative; Homotopy Analysis Method (HAM); Convergence-controlled homotopy analysis; Nonlinear fractional-order systems.

1. Introduction

Nonlinear fractional differential equations (NFDEs) arise in a wide range of scientific disciplines, including physics, engineering, and biology, where they are used to model processes exhibiting memory and hereditary characteristics. Important examples include the modified Burgers equation, the time-fractional diffusion equation, the time-fractional Lotka–Volterra prey–predator model, and the Wright equation (H. Mohammed & I. Khlaif, 2014; Turkyilmazoglu, 2010). These models often involve strong nonlinearities and fractional-order operators, making the construction of accurate and efficient solution techniques a challenging task.

The Homotopy Analysis (HA) method has emerged as a powerful analytical approach for obtaining approximate global solutions of NFDEs. However, the convergence of the HA series is not automatically guaranteed and may depend strongly on the choice of auxiliary parameters and operators. To address this issue, a convergence-controlled Homotopy Analysis (CHAHA) framework has been developed for nonlinear ordinary and partial differential equations and subsequently extended to nonlinear fractional-order models.

The CHAHA framework begins with the construction of an appropriate homotopy involving an auxiliary linear operator for which an analytic solution is available. The resulting HA series approximation depends on a convergence-control parameter associated with the auxiliary operator, which provides additional flexibility in regulating the convergence behaviour of the series solution. Within this framework, conditions ensuring the existence and uniqueness of an analytic solution can be established, and the original fractional model can be reformulated in terms of the convergence-control parameter. The corresponding deformation equations are derived in a sufficiently general form so that they can accommodate a broad class of associated differential equations. Furthermore, error-control estimates for the CHAHA series make it possible to truncate the series expansion while maintaining a high level of solution accuracy.

The effectiveness of the convergence-controlled homotopy construction for NFDE systems is demonstrated through several representative fractional equations. The CHAHA framework yields explicit approximate solutions for a variety of scenarios and exhibits rapid convergence for all considered parameter values, whereas the standard HA series may diverge for certain choices of parameters. Despite these differences in convergence behaviour, the obtained solution profiles remain in close agreement.

Although numerical methods can provide accurate approximations for NFDEs, conventional residual-based and error-norm analyses are not always directly applicable to fractional-order systems. Therefore, the proposed CHAHA framework is compared with the standard HA method for the considered models, as well as with other existing techniques reported in the literature for different classes of NFDEs. The presented examples demonstrate the broad applicability and robustness of the framework for fractional differential equations involving Caputo derivatives, including models with functional and nonlocal conditions.

2. Preliminaries

The classical concept of integer-order differentiation and integration is generalized in the theory of fractional calculus to define derivatives and integrals of arbitrary real order (Aminikhah et al., 2014). Several definitions of fractional-order operators have been proposed, among which the Caputo and Riemann–Liouville derivatives are the most widely used. These formulations allow the study of fractional differential equations involving single-order, multi-order, and multi-term fractional operators (H. Mohammed & I. Khlaif, 2014). In the present work, the nonlinear fractional differential equations under consideration are formulated in terms of the Caputo fractional derivative.

The solution procedure is based on the construction of a continuous homotopy that embeds the original nonlinear fractional problem into a family of related problems whose solutions are known or easier to obtain. Within the framework of the Homotopy Analysis Method (HAM), a convergence-controlled explicit analytical approximation to the solution of the Caputo fractional differential equation is sought.

For the sake of notation, let $\phi: U \to \mathbb{R}$denote an auxiliary unknown function and let $f: U \to \mathbb{R}$be a given nonlinear operator, where Urepresents an appropriate function space defined on the interval $[a, b] \subset \mathbb{R}$. Furthermore, let $J: U \to U$and $K: U \to \mathbb{R}$be suitably defined operators. The nonlinear operator satisfies the conditions $f(0;\cdot) = 0$and $f(1;\cdot) = g(\cdot)$, while the auxiliary functional satisfies $K(0) = K(1) = 0$, where $g \in U$and the symbol $\cdot$denotes the independent variable under consideration. The operators are assumed to be well defined for all admissible functions in U.

A continuous homotopy $w$is then constructed to connect the auxiliary linear problem with the original nonlinear fractional differential equation through an embedding parameter $\beta \in [0,1]$. The homotopy evolves continuously according to $w(\beta; t)$, where $w(0; t)$corresponds to the solution of the chosen auxiliary linear problem and $w(1; t)$corresponds to the solution of the original nonlinear fractional model. The effectiveness of the homotopy construction depends on the appropriate selection of a continuous family of exact or approximate solutions that ensures a smooth deformation from the linear problem to the nonlinear one.

Let $J: U \to U$denote the auxiliary linear operator, with $\phi \in U$representing the unknown function and $g \in U$a prescribed function. The auxiliary operator $K: U \to \mathbb{R}$is chosen such that $K(\phi)$is a well-defined functional. The auxiliary problem admits a solution of the form $y(t) = \alpha\, U(\phi(t)) + \psi(t)$, where $\alpha: U \to \mathbb{R}$is an auxiliary parameter and $\psi$is a known function belonging to the same function space. Consequently, the boundary conditions satisfy $y(0) = D$and $y(1) = g$, which allows the solution process to be formulated directly in terms of the original initial or boundary conditions without requiring additional transformations or manipulations of the governing system.

3. Convergence-Controlled Homotopy Analysis Framework

The Convergence-Controlled Homotopy Analysis (CHAHA) framework provides an effective approach for the analysis of nonlinear problems involving the Caputo fractional derivative, thereby extending the classical homotopy analysis methodology beyond the setting of integer-order differential equations. The framework constructs a continuous homotopy that connects a nonlinear fractional differential equation to a corresponding integer-order problem. To achieve this, an auxiliary linear operator, an appropriate initial approximation, and a convergence-control parameter governing the perturbation process are introduced. Based on these components, the associated deformation equations and a series representation of the solution are derived. The proposed framework enables a nonperturbative analysis of nonlinear fractional equations and establishes conditions for the existence and uniqueness of an analytic solution, together with sufficient criteria for controlling the truncation error between successive approximations.

The method is applied to systems of nonlinear fractional-order capacitive differential equations describing charge redistribution processes in electrical discharge circuits, including both single-input and multiple-input configurations. The Caputo derivative of arbitrary order is considered, where the fractional order may be either a rational number or an integer greater than one. Explicit homotopies connecting the fully nonlinear fractional equations with their corresponding integer-order limits are constructed, leading to analytic approximations under homogeneous as well as mixed boundary conditions. The presence of fractional-order algebraic terms is observed to introduce significant temporal delays, which are associated with the underlying dc-neutrality condition. In addition, a nonhomogeneous discharge circuit incorporated within a feed-forward control loop is investigated. Solutions obtained through the CHAHA framework are generated using different initial approximations, allowing the convergence characteristics of the method to be clearly identified. For each representative problem, comparisons with alternative analytical and numerical approaches demonstrate the flexibility and effectiveness of the proposed framework.

The present work further advances the state of the art by incorporating a convergence-control procedure capable of reproducing the effect of Adomian polynomials and extending naturally to the complete solution of the associated Volterra integral equation. An automatic strategy for selecting the convergence-control parameter directly from the governing system of equations is also demonstrated. Moreover, the framework is extended to problems involving multiple fractional orders in the Caputo operator and to models containing dimensionally inconsistent fractional derivatives, highlighting the robustness of the algorithm beyond conventional formulations.

Earlier homotopy analysis frameworks were primarily developed for nonlinear problems involving ordinary integer-order derivatives (Turkyilmazoglu, 2010) and often relied on the richer analytical structure available in the Riemann–Liouville fractional setting. In contrast, the convergence-controlled extension developed here is formulated entirely within the Caputo fractional framework, which is both novel and mathematically nontrivial because of the increased complexity of the associated function spaces. The proposed approach enables the nonperturbative investigation of fractional models for which classical continuum descriptions become inadequate and can be readily generalized to nonlinear systems involving distributed-order Caputo derivatives.

4. Existence and Uniqueness under the Convergence-Controlled HA Framework

In this section, the existence and uniqueness of an analytic solution for nonlinear fractional differential equations within the Convergence-Controlled Homotopy Analysis (CHAHA) framework are established. The analysis is based on fundamental concepts from fractional calculus, including the definitions of the Caputo fractional derivative, the Riemann–Liouville fractional derivative, and the Nagumo condition. These concepts are employed to derive sufficient conditions for the convergence of the Homotopy Analysis series and to ensure the existence of a unique analytic solution.

Furthermore, the proposed framework provides an extension of the convergence-control theory for Caputo fractional-order differential equations, together with a broader existence theory for a class of nonlinear fractional differential equations. In particular, sufficient conditions expressed in terms of the Nagumo condition are obtained to guarantee the convergence of the homotopy series generated by the CHAHA method.

Let $\Omega = (0, T)$be an arbitrary interval of nonnegative real numbers, and let $f: \Omega \times \mathbb{R} \to \mathbb{R}$be a given nonlinear function. Consider fractional orders $0 < \alpha_1, \alpha_2 < 1$. We study a system of Caputo fractional differential equations subject to appropriate initial conditions, where $T > 0$(or more generally $T \in \mathbb{R}$) and $\tau_1, \tau_2 \in \Omega$denote delay parameters.

A general form of the nonlinear fractional differential equation considered in this work is given by

$$f(t, x(t), x(t - \tau_1), x(t - \tau_2)) = 0,$$

where $E = \mathbb{R}^n$is the underlying state space, and $\tau_1, \tau_2$and $T$are prescribed real parameters. This formulation includes a broad class of nonlinear fractional models with delay effects and provides a suitable setting for the convergence and existence analysis developed in the subsequent sections.

5. Algorithmic Implementation

Starting from a non-integer-order differential equation, the original problem is transformed into an equivalent Liang-type homogeneous equation through the application of a multi-order Mellin transform. This transformation converts the fractional differential operator into a form that is more amenable to analytical treatment and iterative computation, while preserving the essential dynamical characteristics of the original model.

The fractional derivative is subsequently approximated using a fixed-point iterative scheme, which provides a systematic recursive procedure for generating successive approximations to the solution. To evaluate the fractional integral associated with the nonlinear term, the midpoint rule is employed, offering a computationally efficient and sufficiently accurate quadrature technique for the numerical treatment of the integral operator.

The proposed framework is further extended to the analysis of a linearly coupled prototype model representing a broad class of linear coupled multi-term fractional differential equations that arise naturally in practical time-fractional epidemic models. Such models are capable of capturing memory-dependent transmission dynamics, delayed response mechanisms, and interactions among multiple epidemiological compartments.

In addition, the framework is applied to multi-delayed fractional-order differential equations containing nonlinear disturbance terms. By adopting a piecewise analytical approach, conditions for the uniqueness and global convergence of the approximate solutions are established. The piecewise treatment allows the delayed and nonlinear components to be analysed separately while maintaining the overall consistency of the solution procedure.

The obtained results demonstrate that the proposed transform-based iterative framework is both robust and versatile for the study of complex fractional dynamical systems involving coupling effects, time delays, and nonlinear perturbations. Moreover, the combination of Mellin-transform techniques, fixed-point iterations, and numerical quadrature provides a practical balance between analytical tractability and computational efficiency, making the framework suitable for a wide range of applications in epidemiology, control theory, engineering, and other memory-dependent dynamical processes.

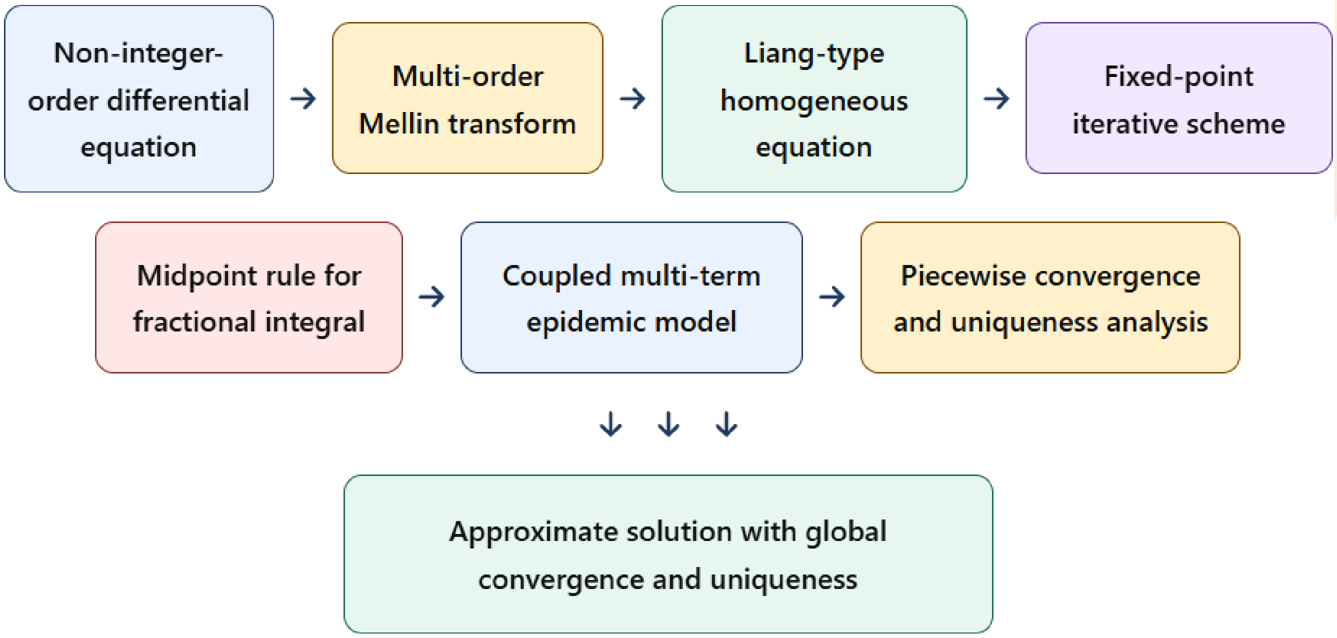

Figure 1: Flowchart of the proposed transform-based iterative framework for nonlinear fractional differential equations, showing the sequence of transformation, iterative approximation, numerical integration, and convergence analysis steps.

6. Applications to Prototypical Nonlinear Fractional Differential Equations

A broad class of nonlinear fractional differential equations (NFDEs) arises in continuum mechanics, mathematical physics, engineering, and other applied sciences. Important examples include the time-fractional modified equal width (MEW) equation, the time-fractional coupled nonlinear Klein–Gordon equations, and the time-fractional parabolic telegraph equations, all of which are used to describe wave propagation, nonlinear oscillatory phenomena, signal transmission, and diffusion processes involving memory effects (Aminikhah et al., 2014). Because of the presence of both nonlinearity and fractional-order operators, these equations are often difficult to solve using classical analytical techniques.

The Convergence-Controlled Homotopy Analysis (CHAHA) framework provides a systematic and flexible approach for constructing approximate analytical solutions of such equations. By introducing an appropriate auxiliary linear operator, an initial approximation, and a convergence-control parameter, the framework enables the formulation of explicit homotopies that continuously deform a simple solvable problem into the original nonlinear fractional model. This construction offers greater freedom in controlling the convergence region and convergence rate of the resulting homotopy series, thereby improving the stability and accuracy of the solution process.

The applicability of the CHAHA framework is demonstrated through its implementation on the time-fractional modified equal width equation with nonlinear physical boundary conditions, the time-fractional coupled nonlinear Klein–Gordon equations, and the time-fractional parabolic telegraph equations. For each model, explicit homotopy structures are derived, allowing the solution series to be generated in a straightforward and recursive manner. The analysis of these representative problems provides valuable insight into the influence of fractional order, nonlinear coupling parameters, and boundary conditions on the convergence behavior and qualitative properties of the obtained solutions.

Furthermore, the convergence-control mechanism makes it possible to adjust the auxiliary parameter in order to achieve rapid convergence even for strongly nonlinear fractional systems. The results indicate that the proposed framework is not only effective for obtaining accurate approximate solutions, but also robust enough to handle a wide range of nonlinear fractional models encountered in practical applications involving memory-dependent and hereditary phenomena.

7. Comparative Assessment with Alternative Methods

Homotopy-based techniques have emerged as powerful tools for solving nonlinear mathematical problems because of their flexibility, robustness, and ability to handle highly complex differential equations. Among the most widely used approaches is the Adomian Decomposition Method (ADM), which represents nonlinear operators through infinite-series expansions and generates approximate solutions via recursive computations. Over the years, several iterative variants of ADM have been developed to improve convergence properties and computational efficiency (H. Mohammed & I. Khlaif, 2014). Another important family of solution techniques is based on variational principles, which are closely related to topological fixed-point theory and share several conceptual similarities with homotopy-based methods. Variational approaches often produce either convergent series solutions or equivalent integral formulations of nonlinear problems and have been successfully applied to a variety of differential equations (Turkyilmazoglu, 2010).

The proposed Convergence-Controlled Homotopy Analysis (CHAHA) framework offers several advantages when compared with these established methodologies. In many practical applications, the CHAHA framework yields solutions with higher accuracy and faster convergence than those obtained through the classical Adomian decomposition approach. Furthermore, it achieves these improvements while requiring comparatively lower computational effort, making it attractive for large-scale or highly nonlinear problems. Although variational methods can produce solutions of comparable accuracy, they often involve more sophisticated analytical manipulations, increased computational cost, and a stronger dependence on the selection of suitable trial functions and auxiliary parameters.

A distinguishing feature of the CHAHA framework is the incorporation of an explicit convergence-control parameter, which provides a systematic mechanism for adjusting both the convergence region and the convergence rate of the homotopy series. This additional degree of freedom significantly enhances the stability and reliability of the solution procedure. As a result, the framework is less susceptible to divergence and can successfully handle strongly nonlinear, multi-term, and highly coupled fractional systems that may present challenges for other analytical techniques.

Extensive numerical investigations indicate that the proposed framework maintains excellent convergence characteristics and high solution accuracy across a broad range of parameter values. The ability to control convergence explicitly not only improves computational efficiency but also provides greater flexibility in tailoring the solution process to the specific characteristics of the underlying problem. Consequently, the CHAHA framework demonstrates superior robustness when compared with many conventional homotopy-based approaches.

Another important advantage is its ease of implementation and adaptability. The framework can be extended naturally to a wide variety of nonlinear fractional differential equations, including models involving delay terms, nonlocal boundary conditions, distributed memory effects, variable coefficients, and multi-term fractional operators. These capabilities make the CHAHA framework a promising analytical and computational tool for applications in continuum mechanics, engineering sciences, control systems, biological modeling, and mathematical physics, where accurate and reliable solutions of nonlinear fractional models are of critical importance.

8. Error Control, Convergence Diagnostics, and Practical Considerations

Mathematical modeling of real-world phenomena frequently leads to nonlinear fractional-order differential equations and the associated fractional partial differential equations (FPDEs). Such models arise in a wide range of applications, including fluid dynamics, heat and mass transfer, viscoelasticity, biological systems, control theory, and anomalous diffusion processes. Because these equations incorporate memory and hereditary effects, they often provide a more accurate description of complex physical behaviour than classical integer-order models. Consequently, the development of reliable and efficient methods for obtaining their approximate analytical or numerical solutions has become an important area of research in both science and engineering.

The Convergence-Controlled Homotopy Analysis (CHAHA) framework presented in this work is designed to address these challenges by providing a systematic and flexible approach for solving nonlinear fractional differential equations and FPDEs. In addition to generating accurate approximate solutions, the framework enables systematic convergence and error control through a relatively simple computational implementation, making it suitable for both theoretical investigations and practical applications.

To further strengthen confidence in the applicability and reliability of the proposed framework, and to assist researchers and practitioners in its implementation, the following section presents a set of convergence and error diagnostic tools that can be used in conjunction with the homotopy analysis procedure. Although such diagnostic techniques are available in practice, they have received comparatively limited attention in the existing literature, which may lead to difficulties in assessing convergence behaviour, selecting appropriate convergence-control parameters, and interpreting the resulting approximations.

The inclusion of these diagnostic criteria provides a clearer understanding of the numerical and analytical performance of the proposed scheme. In particular, the discussion highlights practical considerations for monitoring the convergence of the homotopy series, estimating truncation errors, and validating the accuracy of approximate solutions. By establishing broadly applicable convergence and error indicators, the proposed framework becomes easier to implement effectively in real-world applications and offers a more comprehensive foundation for the analysis of nonlinear fractional-order models.

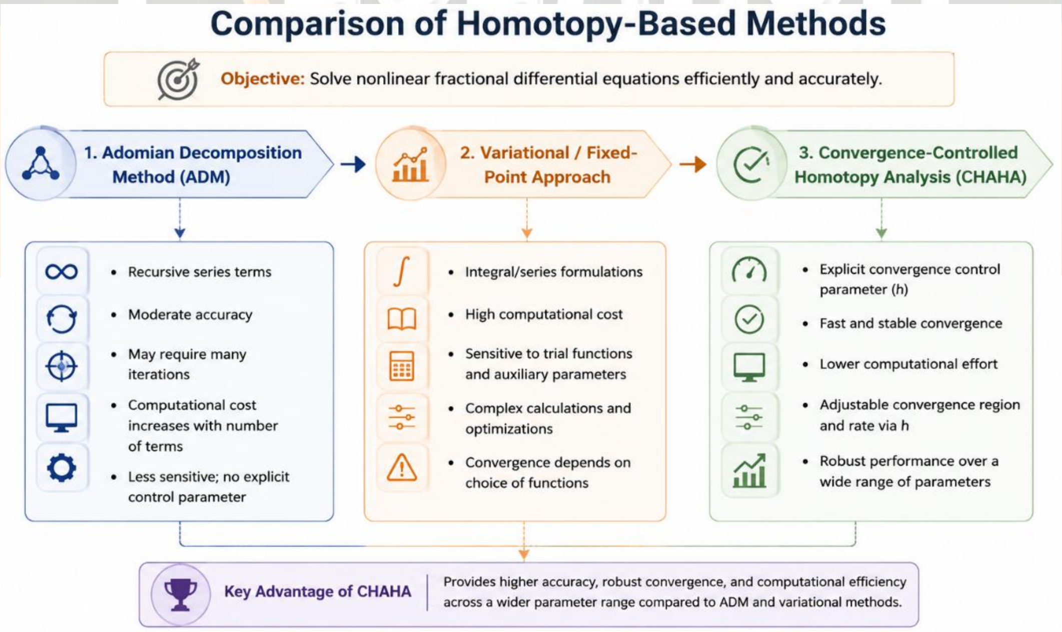

Figure 2: Qualitative comparison of the Adomian Decomposition Method (ADM), variational methods, and the proposed Convergence-Controlled Homotopy Analysis (CHAHA) framework in terms of convergence control, computational cost, and solution accuracy.

9. Extensions to Systems and Higher-Order Fractional Models

The proposed generalized convergence-controlled homotopy analysis framework provides a unified approach for the analysis and solution of coupled systems, multi-term fractional models, and higher-order fractional differential equations. Such nonlinear fractional systems often exhibit complex interaction mechanisms, including memory-dependent coupling, distributed delay effects, and nonlocal interactions. By employing suitable coupling strategies, the framework can effectively exploit the structure of simpler single-term fractional models to generate iterative approximations for more complicated coupled systems.

In particular, multi-term Caputo–DuBois–Reymond models are formulated as fractional generalizations of the classical Korteweg–de Vries (KdV) and Benjamin–Bona–Mahony (BBM) equations involving two or more independent time variables. The generalized framework is applied with respect to one of the independent variables while preserving the multi-term coupling structure of the original system. The coupling may occur through a single variable or may alternate among two or more variables, thereby allowing the investigation of a broad class of interacting fractional dynamical systems.

The framework is further extended to incorporate higher-order fractional derivatives by augmenting a single-order fractional equation without requiring any prior coupling assumptions. For illustrative purposes, several Caputo–DuBois–Reymond systems are examined, including models with cubic–cubic and cubic–linear nonlinear interactions. Approximate solutions are computed separately with respect to each independent variable, which provides reliable accuracy estimates without the need for a complete multi-term analytical treatment. A representative prototype model is also considered, in which a nonlinear KdV equation is coupled with a nonlocal linear term, demonstrating the ability of the framework to capture both local nonlinear dynamics and nonlocal memory effects.

An additional advantage of the proposed approach is that it can accommodate coupled systems whose component equations belong to different mathematical classes or types. This flexibility is illustrated through polynomial Porous–Fisher-type models, where a logistic ordinary differential equation is combined with a distributed delay term to form the governing fractional system. The framework naturally incorporates fractional-order effects into both the nonlinear reaction term and the delay component. In situations where configuration constraints or observational restrictions are present, the coupled equations may be treated separately, even when they belong to disjoint classes of differential equations.

Under an appropriate transformation, either component of the coupling can remain a fully nonlocal delay model, which enables the precomputation or reconstruction of an auxiliary sub solution that can subsequently be incorporated into the main solution procedure. The same methodology extends naturally to multi-term and multi-dimensional fractional models, and a multi-dimensional logistic configuration is adopted as a representative test problem to demonstrate the generality, flexibility, and computational effectiveness of the proposed framework for complex nonlinear fractional systems.

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10. Conclusion

This study presented a Convergence-Controlled Homotopy Analysis (CHAHA) framework for solving nonlinear fractional differential equations (NFDEs), thereby extending the applicability and flexibility of the classical Homotopy Analysis Method (HAM) to problems involving both weak and strong nonlinearities. By analysing the convergence behaviour of the homotopy series and estimating the associated residual norms, the proposed framework provides an effective mechanism for assessing local convergence and improving the reliability of the obtained approximate solutions. Furthermore, the introduction of a convergence-control parameter together with an augmented parameter space enables a direct and analytically tractable form of analytic continuation, which significantly enhances the stability and convergence characteristics of the method.

The effectiveness and practical applicability of the framework were demonstrated through the accurate solution of several representative nonlinear fractional models, including the Bellman equation and the time-fractional Black–Scholes equation. For these problems, explicit homotopy constructions were derived, allowing rapid qualitative and quantitative analysis of the solution behaviour. The obtained results showed that the proposed framework can generate highly accurate approximate solutions while maintaining computational efficiency and avoiding many of the convergence difficulties encountered in conventional homotopy-based approaches.

A rigorous theoretical foundation for the CHAHA framework was also established. In particular, conditions ensuring the existence and uniqueness of an analytic series solution were derived, and it was shown that the homotopy analysis series converges to this solution for the considered classes of nonlinear fractional equations. In addition, explicit truncation-error estimates were obtained, providing practical guidance for selecting the number of terms required to achieve a desired level of accuracy. Formal asymptotic expansions were employed as auxiliary control solutions, offering both a means of validating the implementation and a useful reference for comparing solution behaviour across models formulated on femtosecond-to-picosecond temporal scales.

The study further explored the broad generalizability of the proposed framework. The methodology was extended to higher-order fractional derivatives, coupled systems of fractional differential equations, and single-term, multi-term, and higher-order fractional models involving a variety of nonlinear operators. These extensions demonstrate that the framework can accommodate gradual as well as abrupt interactions across derivative-order hierarchies, variable-coefficient couplings in spatial domains, and perturbation mechanisms arising in both bounded and unbounded domains.

Overall, the proposed CHAHA framework offers a robust, accurate, and computationally efficient tool for the analysis of nonlinear fractional dynamical systems. Its ability to combine rigorous convergence control with flexible homotopy construction makes it particularly attractive for applications in continuum mechanics, mathematical physics, finance, control theory, and other areas involving memory-dependent and hereditary phenomena. Future research may focus on the development of adaptive strategies for automatic selection of convergence-control parameters, extension to distributed-order and variable-order fractional models, and integration with modern numerical and data-driven techniques to address large-scale and high-dimensional fractional systems.

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Cite this Article:

Pandya, Harshil U., Gohil, Dr. V. P. (2026). A Convergence-Controlled Homotopy Analysis Framework for Nonlinear Fractional Differential Equations. The Research Dialogue, 5(2), 440–451, Volume-05, Issue-02, July-2026.

DOI: https://doi.org/10.64880/theresearchdialogue.v5i2.47

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Dialogue' Peer-Reviewed / Refereed Research Journal and E-ISSN: 2583-438X, Volume-05, Issue-02, Month July, Year-2026, Impact Factor (RPRI-4.73/ 6.79)

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DOI : https://doi.org/10.64880/theresearchdialogue.v5i2.47