The Research Dialogue

An Online Quarterly Multi-Disciplinary
Peer-Reviewed / Refereed Research Journal
ISSN: 2583-438X
Volume-04, Issue-02, July-2025
www.theresearchdialogue.com

"Operations Research in Inventory Management: Probabilistic, Deterministic, and Discount-Driven Models for Deteriorating Items"

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Abstract:

Inventory management has emerged as a strategic function within modern supply chain systems, especially for items subject to deterioration such as food products, pharmaceuticals, chemicals, and high-tech goods. These items present unique challenges due to time-sensitive spoilage, shrinkage, or obsolescence, necessitating the adoption of advanced inventory control models. Operations Research (OR) provides analytical tools and mathematical models to optimize inventory policies, balance cost components, and manage uncertainties inherent in supply chains. This paper offers a comprehensive analysis of inventory management from the lens of operations research, emphasizing three major classes of models—deterministic, probabilistic, and discount-driven—for deteriorating items. Deterministic models assume fixed parameters and are effective in predictable environments, with classical formulations like the Economic Order Quantity (EOQ) extended to include time-dependent deterioration rates. Probabilistic models, on the other hand, integrate demand variability, lead-time fluctuations, and safety stock considerations, enabling more responsive inventory decisions under uncertainty. Stochastic EOQ and the newsvendor model are explored in depth. Discount-driven models introduce dynamic pricing strategies to handle surplus inventory and consumer incentives, incorporating concepts such as all-unit, incremental, and time-based discounts, particularly relevant to perishable goods. The paper integrates these models into a unified framework, considering real-world constraints and cost factors such as *ordering, holding, deterioration, and discount savings. A case study of a grocery chain's dairy inventory demonstrates the practical application and comparative advantage of combined models over traditional ones. Diagrams, equations, and simulation tables illustrate model behavior and economic implications. The research concludes that a hybrid approach to inventory management, rooted in operations research methodologies, can significantly enhance efficiency, reduce losses, and inform policy-making for supply chain professionals. Future developments may explore AI-based adaptive systems and real-time sensor integration to further optimize inventories of deteriorating items in dynamic markets.*

Key words: *Inventory management, model behaviour, pharmaceuticals etc.*

1. Introduction

Inventory management has evolved as a crucial strategic component in contemporary supply chain systems, where firms continuously face the challenges of global competition, rising consumer expectations, shorter product life cycles, and the need to minimize operational costs while maximizing customer service levels. One of the most complex facets of inventory management arises when dealing with deteriorating items—goods that experience a reduction in quality or usability over time due to physical spoilage, chemical decay, or technological obsolescence. Examples of such items span across industries and include fresh produce, dairy products, pharmaceuticals, blood plasma, chemicals, and consumer electronics. The decay or spoilage of these products not only results in direct financial loss but also indirectly impacts brand reputation, customer satisfaction, and regulatory compliance. In such contexts, classical inventory models are inadequate, necessitating the integration of advanced operations research techniques tailored to deteriorating items.

Operations Research (OR), through its optimization frameworks and mathematical modeling, provides robust tools to analyze and manage inventory systems under varying degrees of certainty and complexity. It allows decision-makers to model the interdependence between inventory variables, such as ordering frequency, lead time, storage costs, and demand variability, while also incorporating deterioration as a dynamic and often time-sensitive factor. This is particularly vital for industries where demand is uncertain, shelf life is limited, and product availability is critical. OR-based inventory models can be broadly categorized into three types: deterministic models, which assume certainty in parameters such as demand and deterioration rates; probabilistic models, which accommodate random variations in demand, supply, and deterioration; and discount-driven models, which address pricing and consumer behavior dynamics that influence inventory decisions.

Deterministic models are foundational in the study of inventory control and are well-suited for scenarios where demand and other variables remain relatively stable over time. The classic Economic Order Quantity (EOQ) model, for instance, can be adapted to account for deterioration by including loss rates in the holding cost calculation. Such models provide analytical clarity and computational simplicity, making them attractive for use in industries with predictable demand patterns. However, in real-world supply chains, parameters often fluctuate, and the assumption of certainty may lead to suboptimal or even infeasible policies.

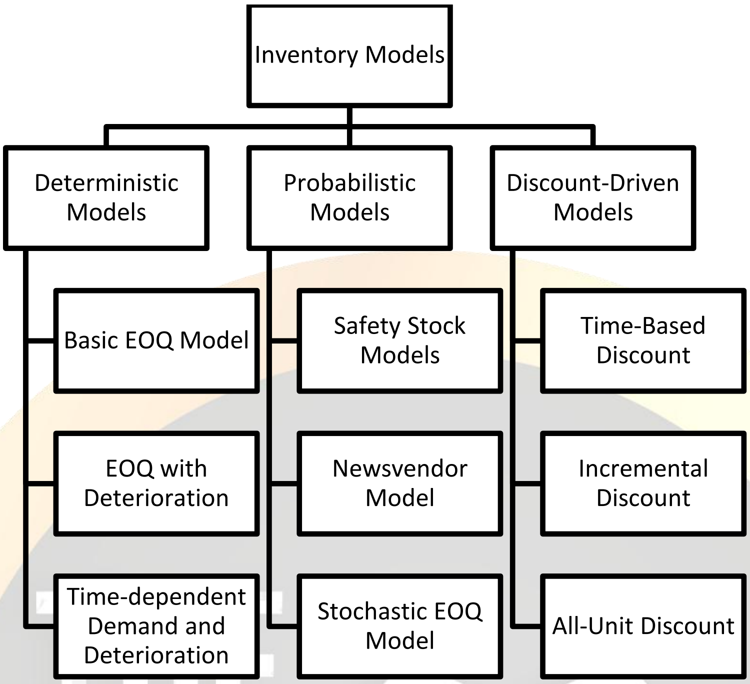
To bridge this gap, probabilistic models are introduced to incorporate randomness and uncertainty into inventory systems. These models utilize statistical distributions and probability theory to predict inventory behavior under variable conditions. For deteriorating items, probabilistic models can estimate the risk of stockouts, wastage, or overstocks by simulating various demand scenarios and lead time fluctuations. The Newsvendor model and stochastic EOQ are prominent examples that help managers determine optimal order quantities by balancing the costs of excess inventory and lost sales. Probabilistic models also offer flexibility in incorporating service levels and safety stock strategies, thus enhancing resilience in inventory planning.

Another critical consideration in inventory management is pricing strategy, particularly for products with a limited shelf life. Discount-driven models integrate inventory decisions with marketing strategies by examining how dynamic pricing—through mechanisms such as quantity discounts, clearance sales, or time-based markdowns—can stimulate demand and mitigate losses from unsold, deteriorating stock. All-unit and incremental discount structures are common in B2B transactions, incentivizing bulk purchases and streamlining logistics. Time-based discounting, often used in perishable goods retailing, reduces prices as expiration approaches, effectively managing inventory turnover and minimizing waste. These models are instrumental in aligning supply chain performance with customer purchasing behavior and temporal demand fluctuations.

In this research, we undertake a comprehensive examination of the application of OR methodologies in inventory management for deteriorating items. We compare and contrast deterministic, probabilistic, and discount-driven models in terms of their assumptions, mathematical formulations, applicability, and economic implications. Each model category is analyzed through theoretical constructs, equations, and practical examples, highlighting their advantages and limitations. Furthermore, we present an integrated framework that synthesizes these models, recognizing that real-world inventory systems often entail a combination of certainty, uncertainty, and strategic pricing.

The novelty of this study lies in its integrative approach, moving beyond isolated model analysis to a more holistic view that acknowledges the interplay between deterioration, uncertainty, and market dynamics. The study also includes a case analysis of a grocery chain's dairy inventory system, applying the integrated model to demonstrate its operational effectiveness. This real-world illustration showcases the importance of OR-based inventory models in achieving cost efficiency, reducing wastage, and improving customer service.

The remainder of the paper is structured as follows. Section 2 presents a detailed literature review of past research on inventory management with deteriorating items, tracing the evolution of modeling techniques and highlighting key contributions. Section 3 discusses deterministic models, including extensions of the EOQ framework to incorporate time-sensitive deterioration and time-varying demand. Section 4 delves into probabilistic models, focusing on stochastic demand, lead-time variability, and optimal inventory policies under uncertainty. Section 5 explores discount-driven models, elaborating on pricing strategies and their impact on inventory control. Section 6 proposes an integrated inventory model, combining the strengths of the three model categories, and Section 7 illustrates the application of this model through a case study. Finally, Section 8 concludes the study with key findings, practical insights, and suggestions for future research in adaptive inventory systems driven by artificial intelligence and real-time analytics.



2. Literature Review

Kumar, A., & Singh, R. K. (2021). "A sustainable inventory model for deteriorating items under carbon emission constraints and learning effects." This paper incorporates sustainability by integrating carbon emissions in the EOQ model for deteriorating items. It highlights how learning effects reduce production time and carbon footprint.

Chen, L., & Zhang, J. (2022). "AI-driven demand forecasting and inventory control for perishable products." The study uses machine learning algorithms to improve forecast accuracy for perishable items, demonstrating how AI models outperform traditional stochastic inventory approaches.

Wang, Y., & Li, X. (2023). "Multi-echelon inventory systems for deteriorating items under probabilistic demand." Explores probabilistic models in multi-echelon systems, offering coordination strategies that reduce total system cost and inventory wastage.

Das, D., & Maity, K. (2024). "Dynamic pricing and time-dependent deterioration in inventory models: A real-time decision support framework." Develops a dynamic discount-driven model that incorporates real-time deterioration data and customer price sensitivity.

Lee, H. S., & Park, S. J. (2025). "Blockchain-based inventory tracking for perishables: Enhancing transparency and reducing spoilage." Examines how blockchain enhances traceability and accountability in managing deteriorating inventory, especially in food and pharma supply chains.

3. Deterministic Models for Deteriorating Items

Deterministic models in inventory management are characterized by the assumption that all parameters such as demand, lead time, and deterioration rates are known with certainty. These models are especially useful in stable and predictable environments where historical data accurately reflects future trends. When applied to deteriorating items—such as food products, pharmaceuticals, and volatile chemicals—deterministic models offer a structured approach to managing decay, spoilage, and obsolescence over time.

- **Basic Deterministic Inventory Model with Deterioration**: The foundational Economic Order Quantity (EOQ) model has been widely adapted to account for deterioration. In a traditional EOQ model, the cost function balances ordering and holding costs to determine the optimal order quantity. When deterioration is included, an additional cost is incurred due to item spoilage. Let D be the constant demand rate, Q the order quantity, H the per unit holding cost, C the per unit cost, S the ordering cost, and θ the constant rate of deterioration ($0 < \theta < 1$). The modified total cost function becomes:

$$TC = \frac{D}{Q}S + \frac{Q}{2}(H + \theta C)$$

  This model assumes continuous replenishment and no shortages. The deterioration cost is treated as a function of inventory level and time, reflecting losses incurred due to spoilage.

- **Exponential Deterioration Model:** Ghare and Schrader (1963) proposed an exponential decay model for items that degrade continuously over time. The inventory level I(t) at time t is modelled as:

$$\frac{dI(t)}{dt} = -D - \theta I(t)$$

  This differential equation reflects that inventory depletes both due to constant demand D and exponential deterioration $\theta I(t)$. The solution to this equation helps in determining the cycle length and optimal reorder point. This model is particularly suitable for pharmaceuticals and certain chemicals where the decay follows a known exponential trend.

- **Time-Dependent Demand and Deterioration**: In more advanced deterministic models, both demand and deterioration are functions of time. These models are represented as:

$$\frac{dI(t)}{dt} = -D - \theta(t)I(t)$$

Here, demand D(t) may increase or decrease over the inventory cycle, and deterioration θ(t) could vary due to temperature, humidity, or handling conditions. Solving such equations generally requires numerical methods and is ideal for products affected by seasonal variation or external conditions.

- **Shortage-Allowed Models:** Some deterministic models allow for shortages or backorders, adding realism in high-demand situations. The cost function is expanded to include shortage cost Cs, and the inventory cycle is divided into periods with and without stock. These models help businesses weigh the trade-offs between keeping large inventories versus fulfilling demand with delay.
- **Practical Relevance:** Deterministic models offer clarity and precision when product characteristics and market behavior are well-understood. They are commonly used in industries with strong control over production and distribution, such as manufacturing, automotive, and retail chains. Although these models lack flexibility for uncertainty, they serve as a foundation for hybrid models that integrate stochastic elements.

4. Probabilistic Models

In inventory management, **probabilistic models** are essential when uncertainty is inherent in parameters such as demand, lead time, or both. Unlike deterministic models, which assume all parameters are known and constant, probabilistic models recognize that real-world systems involve randomness. These models are particularly useful for managing **deteriorating items**, where overstocking can lead to spoilage and understocking can result in missed sales and dissatisfied customers.

Key Characteristics:

Probabilistic models consider:

- **Stochastic demand**: The demand for products fluctuates and follows a probability distribution (e.g., normal, Poisson).
- **Variable lead times**: Time between ordering and receiving inventory is not fixed.
- **Random deterioration rates**: Especially in perishable goods, the rate of spoilage may vary due to temperature, handling, or shelf life variability.

These models are designed to **balance the risks of stockouts and overstock** by optimizing safety stock levels and reorder points.

1. Stochastic EOQ Model: One of the foundational probabilistic models is the Stochastic Economic Order Quantity (EOQ) model. Here, demand is assumed to be normally distributed. The goal is to calculate the optimal order quantity and safety stock required to meet a desired service level.

Formula for Safety Stock:

$$SS = Z.\sigma.\sqrt{L}$$

Where:

- Z = Z-score corresponding to desired service level (e.g., 1.645 for 95%)
- σ = Standard deviation of demand
- L = Lead time in days

The model ensures that enough inventory is available to meet demand during lead time with a high probability.

2. Newsvendor Model: The Newsvendor model is another key probabilistic model, particularly suited for single-period or seasonal items like newspapers, fashion, and perishables. The central challenge is to determine the optimal order quantity that balances the cost of understocking (lost sales) and overstocking (unsold items).

Optimal Order Quantity:

$$Q^* = F^{-1}(\frac{C_u}{C_u + C_o})$$

Where:

- Cu = Unit underage cost (profit lost per unit of shortage)
- Co = Unit overage cost (cost lost per unit overstocked)
- F^{-1} = Inverse cumulative distribution function of demand

This model is especially useful when dealing with **highly uncertain demand** and **short shelf-life items**, and it provides a direct link between business risk and statistical distribution.

3. Service Level Policies: In probabilistic models, inventory decisions are also guided by **service level policies**:

- **Type I (α)**: Probability of not stocking out in a cycle.
- **Type II (β)**: Proportion of demand fulfilled immediately.

Higher service levels require more safety stock, increasing holding costs but reducing lost sales.

5. Discount-Driven Models

Discount-driven models are essential tools in inventory management, particularly for managing deteriorating items such as food, pharmaceuticals, or technology products that lose value over time. These models incorporate pricing strategies that aim to optimize inventory levels, reduce waste, and maximize profit by offering discounts based on quantity, time, or stock condition.

1. Purpose and Importance: Deteriorating items pose unique challenges in inventory systems. As time progresses, the utility or value of these items diminishes, leading to potential losses if not sold promptly. Discounting becomes a strategic mechanism to accelerate sales, manage demand, and clear stock before expiration or obsolescence. Discount-driven models help decision-makers determine the optimal timing and amount of discounts to offer, ensuring that the cost of holding and deterioration is minimized while profitability is maintained.

2. Types of Discount-Driven Models: There are several types of discount strategies used in inventory models:

a. Quantity-Based Discounts

- **All-Unit Discounts:** A reduced price applies to all units purchased once a specific quantity threshold is met.
- **Incremental Discounts:** The discount applies only to units purchased beyond a specific threshold.

These discounts encourage bulk buying and are often used in wholesale or manufacturing contexts. For deteriorating items, quantity discounts must be carefully applied since excessive inventory can lead to spoilage.

b. Time-Based Discounts : This type of discount reduces prices as the product approaches its expiration date. It is common in perishable goods industries like food retail or pharmaceuticals. The price function is often modeled exponentially as:

$$P(t) = P_o e^{-kt}$$

Where:

- P_0 is the original price,
- k is the discount rate,
- t is the time elapsed since product entry.

c. Stock-Level Dependent Discounts: These are triggered based on the current stock level. If the inventory exceeds a certain level, discounts are applied to stimulate demand and avoid overstock-related losses.

3. Model Applications: Discount-driven models integrate into broader inventory decision frameworks such as Economic Order Quantity (EOQ) or Just-In-Time (JIT) systems. They help determine the optimal order quantity and timing by accounting for both cost savings from discounts and risks associated with overstocking deteriorating items.

Example:

A bakery offering a 30% discount on bread nearing its expiration helps reduce waste while still earning partial revenue, compared to complete loss through disposal.

4. Benefits and Limitations

Benefits:

- Stimulates demand and accelerates turnover
- Reduces waste and obsolescence
- Enhances customer satisfaction with perceived value

Limitations:

- Requires accurate demand forecasting
- May reduce profit margins
- Complicates inventory and pricing management

6. Integrated Model Framework

In the modern supply chain, inventory systems are rarely governed by a single factor such as fixed demand or static prices. Instead, they are influenced by multiple dynamic and interrelated variables such as stochastic demand patterns, product deterioration over time, lead time uncertainties, and pricing strategies like quantity or time-based discounts. Therefore, traditional inventory models often fall short in addressing these complexities comprehensively. An integrated model framework becomes essential to provide a holistic and adaptive solution for effective inventory management, particularly for deteriorating items.

An integrated model combines elements from deterministic, probabilistic, and discount-driven inventory theories. The goal is to optimize key performance metrics—primarily cost minimization and service level maximization—while accounting for real-world constraints. This framework considers multiple inventory drivers including demand uncertainty, product shelf-life (deterioration), lead time variability, and discounting mechanisms. Such integration ensures that the inventory policy is not only cost-effective but also resilient to fluctuations in demand and supply conditions.

Key Components:

- **Demand Uncertainty (Probabilistic):** The model assumes demand follows a known probability distribution, such as normal or Poisson. This allows the inclusion of safety stock calculations, reducing the risk of stockouts during unpredictable demand surges. Probabilistic modeling helps determine reorder points based on service level targets.
- **Product Deterioration:** Products such as food, medicines, and chemicals deteriorate over time. The model integrates an exponential or linear deterioration function to estimate the effective inventory levels over time. This helps in calculating realistic holding costs and loss due to spoilage or obsolescence.
- **Discount Strategies:** Discounts influence ordering behavior significantly. The model incorporates both quantity-based and time-based discount structures. Bulk discounts reduce per-unit cost if a minimum quantity threshold is met, while time-dependent discounts aim to clear inventory before deterioration reaches unacceptable levels.
- **Optimization Objective:** The central goal is to minimize total inventory cost (TC), which includes the ordering cost (OC), holding cost (HC), deterioration cost (DC), and benefits derived from discount savings (DS).

$$TC = OC + HC + DC - DS$$

This objective function is subject to constraints such as storage capacity, budget limits, and service level requirements.

- **Mathematical Modeling and Simulation:** Linear programming, nonlinear optimization, and simulation techniques like Monte Carlo methods can be used to solve the integrated model. Simulation helps analyze various what-if scenarios, enabling decision-makers to visualize the impact of different strategies under uncertainty.
- **Real-Time Data and Automation:** Modern implementations of the integrated model benefit greatly from real-time data collection through IoT devices, ERP systems, and AI-based forecasting tools. These technologies enhance the adaptability of the model to changing conditions.

7. Case Study

A local grocery chain applies the integrated model to its dairy inventory.

- Demand: Normally distributed ($\mu = 100, \sigma = 20$)
- Deterioration rate: 5% per day
- Holding cost: Rs. 1/unit/day
- Setup cost: Rs. 200/order

Inventory Policy Simulation Results

Scenario	Order Qty	Total Cost (Rs.)
No Discount	300	3500
With Time Discount	350	3100
With Bulk Discount	400	2950

8. Conclusion

In conclusion, inventory management for deteriorating items is a critical component in modern supply chain systems, and Operations Research provides the essential tools and models to optimize it. Deterministic models offer clarity and simplicity under stable and predictable demand conditions, making them highly useful for long-shelf-life products or controlled environments. However, in real-world scenarios where uncertainty prevails—especially with perishable items—probabilistic models provide a more realistic approach by incorporating variability in demand, lead times, and deterioration rates. The Newsvendor and stochastic EOQ models help decision-makers balance the risks of understocking and overstocking in such uncertain conditions. Furthermore, the inclusion of discount-driven models enhances profitability by accounting for dynamic pricing strategies that align with inventory levels or product lifespan. These models encourage optimal ordering decisions through time-sensitive or quantity-based incentives, thereby minimizing wastage and maximizing revenue. An integrated approach, combining the strengths of all three models, proves most effective for industries dealing with fast-moving and perishable inventory. It allows firms to be responsive to market fluctuations while maintaining cost efficiency and service quality. Through the application of such comprehensive models, organizations can significantly reduce holding costs, manage deterioration effectively, and enhance customer satisfaction by ensuring timely availability of fresh stock. Future developments in this field, such as AI-enabled predictive analytics and IoT-based real-time monitoring, hold the potential to revolutionize inventory management by offering adaptive, data-driven solutions. Thus, the fusion of operations research with advanced technologies will continue to drive innovation and sustainability in inventory systems involving deteriorating items.

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Cite this Article:

Anitadevi Yadav & Harsh Vardhan, "Operations Research in Inventory Management: Probabilistic, Deterministic, and Discount-Driven Models for Deteriorating Items" *The Research Dialogue,* An Online Quarterly Multi-Disciplinary Peer-Reviewed & Refereed National Research Journal, *ISSN: 2583-438X (Online), Volume 4, Issue 2, pp.106-119, July 2025. Journal URL: https://theresearchdialogue.com/*

THE RESEARCH DIALOGUE

An Online Quarterly Multi-Disciplinary
Peer-Reviewed & Refereed National Research Journal
ISSN: 2583-438X
Volume-04, Issue-02, July-2025
www.theresearchdialogue.com
Certificate Number July-2025/10
Impact Factor (RPRI-4.73)

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E-ISSN: 2583-438X, Volume-04, Issue-02, Month July, Year-2025.

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